

Line and Plane

EXERCISE 6.1 [PAGES 200 - 201]

Exercise 6.1 | Q 1 | Page 200

Find the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to vector $4\hat{i} - \hat{j} + 2\hat{k}$.

Solution:

The vector equation of the line passing through A(\vec{a}) and parallel to the vector \vec{b} is $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar.

\therefore the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to the vector $4\hat{i} - \hat{j} + 2\hat{k}$ is
$$\vec{r} = (-2\hat{i} + \hat{j} + \hat{k}) + \lambda(4\hat{i} - \hat{j} + 2\hat{k}).$$

Exercise 6.1 | Q 2 | Page 200

Find the vector equation of the line passing through points having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the line passing through the A(\vec{a}) and B(\vec{b}) is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, λ is a scalar.

\therefore the vector equation of the line passing through the points having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$ is

$$\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda[(6\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} + 4\hat{j} - 7\hat{k})]$$

i.e. $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(3\hat{i} - 5\hat{j} + 8\hat{k}).$

Exercise 6.1 | Q 3 | Page 200

Find the vector equation of line passing through the point having position vector $5\hat{i} + 4\hat{j} + 3\hat{k}$ and having direction ratios $-3, 4, 2$.



Solution:

Let A be the point whose position vector is $\vec{a} = 5\hat{i} + 4\hat{j} + 3\hat{k}$.

Let \vec{b} be the vector parallel to the line having direction ratio = -3, 4, 2

Then, $\vec{b} = -3\hat{i} + 4\hat{j} + 2\hat{k}$

The vector equation of the line passing through A(\vec{a}) and parallel to \vec{b} is $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar.

\therefore the required vector equation of the line is

$$\vec{r} = 5\hat{i} + 4\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 4\hat{j} + 2\hat{k}).$$

Exercise 6.1 | Q 4 | Page 200

Find the vector equation of the line passing through the point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.

Solution:

Let $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$

The vector perpendicular to the vectors \vec{b} and \vec{c} is given by

$$\begin{aligned}\vec{b} \times \vec{c} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \\ &= \hat{i}(1+1) - \hat{j}(1-2) + \hat{k}(-1-2) \\ &= 2\hat{i} + \hat{j} - 3\hat{k}\end{aligned}$$

Since the line is perpendicular to the vector \vec{b} and \vec{c} , it is parallel to $\vec{b} \times \vec{c}$.

The vector equation of the line passing through

A(\vec{a}) and parallel to $\vec{b} \times \vec{c}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c})$, where λ is a scalar.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Hence, the vector equation of the required line is $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$.

Exercise 6.1 | Q 5 | Page 200

Find the vector equation of the line passing through the point having position vector $-\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$.

Solution:

Let A be point having position vector $\vec{a} = -\hat{i} - \hat{j} + 2\hat{k}$

The required line is parallel to the line

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$$

\therefore it is parallel to the vector

$$\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$$

The vector equation of the line passing through A(\vec{a}) and parallel to \vec{b} is $\vec{r} = \vec{a} + \lambda\vec{b}$ where λ is a scalar.

\therefore the required vector equation of the line is

$$\vec{r} = (-\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k}).$$

Exercise 6.1 | Q 6 | Page 200

Find the Cartesian equations of the line passing through A(-1, 2, 1) and having direction ratios 2, 3, 1.

Solution: The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

\therefore the cartesian equations of the line passing through the point (-1, 2, 1) and having direction ratios 2, 3, 1 are

$$\frac{x - (-1)}{2} = \frac{y - 2}{3} = \frac{z - 1}{1}$$

$$\text{i.e. } \frac{x + 1}{2} = \frac{y - 2}{3} = \frac{z - 1}{1}.$$

Exercise 6.1 | Q 7 | Page 200

Find the Cartesian equations of the line passing through A(2, 2, 1) and B(1, 3, 0).

Solution: The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Here, $(x_1, y_1, z_1) \equiv (2, 2, 1)$ and $(x_2, y_2, z_2) \equiv (1, 3, 0)$

\therefore the required cartesian equations are

$$\frac{x - 2}{1 - 2} = \frac{y - 2}{3 - 2} = \frac{z - 1}{0 - 1}$$

$$\text{i.e. } \frac{x - 2}{-1} = \frac{y - 2}{1} = \frac{z - 1}{-1}.$$

Exercise 6.1 | Q 8 | Page 200

A(-2, 3, 4), B(1, 1, 2) and C(4, -1, 0) are three points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear.

Solution: We find the cartesian equations of the line AB. The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Here, $(x_1, y_1, z_1) \equiv (-2, 3, 4)$ and $(x_2, y_2, z_2) \equiv (1, 1, 2)$

\therefore the required cartesian equations of the line AB are

$$\frac{x - (-2)}{1 - (-2)} = \frac{y - 3}{1 - 3} = \frac{z - 4}{2 - 4}$$

$$\therefore \frac{x + 2}{3} = \frac{y - 3}{-2} = \frac{z - 4}{-2}$$

$$\therefore \frac{x + 2}{3} = \frac{y - 3}{-2} = \frac{z - 4}{-2}$$

C = (4, -1, 0)

$$\text{For } x = 4, \frac{x + 2}{3} = \frac{4 + 2}{3} = 2$$

For $y = -1$, $\frac{y-3}{-2} = \frac{-1-3}{-2} = 2$

For $z = 0$, $\frac{z-4}{-2} = \frac{0-4}{-2} = 2$

\therefore coordinates of C satisfy the equations of the line AB.

\therefore C lies on the line passing through A and B.

Hence, A, B, C are collinear.

Exercise 6.1 | Q 9 | Page 200

Show that the lines given by

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} \text{ and } \frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$$

intersect. Also, find the coordinates of their point of intersection.

Solution: The equations of the lines are

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} = \lambda \quad \dots(\text{say})\dots(1)$$

$$\text{and } \frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4} = \mu \quad \dots(\text{say})\dots(2)$$

From (1), $x = -1 - 10\lambda$, $y = -3 - \lambda$, $z = 4 + \lambda$

\therefore the coordinates of any point on the line (1) are $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda)$

From (2), $x = -10 - \mu$, $y = -1 - 3\mu$, $z = 1 + 4\mu$

\therefore the coordinates of any point on the line (2) are $(-10 - \mu, -1 - 3\mu, 1 + 4\mu)$

Lines (1) and (2) intersect, if $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda) = (-10 - \mu, -1 - 3\mu, 1 + 4\mu)$

\therefore the equation $-1 - 10\lambda = -10 - \mu$, $-3 - \lambda = -1 - 3\mu$ and $4 + \lambda = 1 + 4\mu$ are simultaneously true.

Solving the first two equations, we get, $\lambda = 1$, and $\mu = 1$.

These values of λ and μ satisfy the third equation also.

\therefore the lines intersect.

Putting $\lambda = 1$ in $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda)$ or $\mu = 1$ in $(-10 - \mu, -1 - 3\mu, 1 + 4\mu)$, we get the point of intersection $(-11, -4, 5)$.

Exercise 6.1 | Q 10 | Page 200

A line passes through $(3, -1, 2)$ and is perpendicular to lines

$\bar{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$. Find its equation.

Solution:

The line $\bar{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ is parallel to the vector $\bar{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and the line $\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ is parallel to the vector $\bar{c} = \hat{i} - 2\hat{j} + 2\hat{k}$.

The vector perpendicular to the vectors \bar{b} and \bar{c} is given by

The vector perpendicular to the vectors \bar{b} and \bar{c} is given by

$$\begin{aligned}\bar{b} \times \bar{c} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix} \\ &= \hat{i}(-4 + 2) - \hat{j}(4 - 1) + \hat{k}(-4 + 2) \\ &= -2\hat{i} - 3\hat{j} - 2\hat{k}\end{aligned}$$

Since the required line is perpendicular to the given lines,

it is perpendicular to both \bar{b} and \bar{c} .

\therefore It is parallel to $\bar{b} \times \bar{c}$

The equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} and \bar{c} is

$\bar{r} = \bar{a} + \lambda(\bar{b} \times \bar{c})$, where λ is a scalar.

Here, $\bar{a} = 3\hat{i} - \hat{j} + 2\hat{k}$

\therefore the equation of the required line is

$$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - 3\hat{j} - 2\hat{k})$$

or

$$\bar{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 2\hat{k}), \text{ where } \mu = -\lambda.$$

Exercise 6.1 | Q 11 | Page 201

Show that the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$ passes through the origin.

Solution:

The equation of the line is

$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$$

The coordinates of the origin O are (0, 0, 0)

$$\text{For } x = 0, \frac{x-2}{1} = \frac{0-2}{1} = -2$$

$$\text{For } y = 0, \frac{y-4}{2} = \frac{0-4}{2} = -2$$

$$\text{For } z = 0, \frac{z+4}{-2} = \frac{0+4}{-2} = -2$$

∴ coordinates of the origin O satisfy the equation of the line.

Hence, the line passes through the origin.

EXERCISE 6.2 [PAGE 207]

Exercise 6.2 | Q 1 | Page 207

Find the length of the perpendicular (2, -3, 1) to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1}.$$

Solution1:

Let PM be the perpendicular drawn from the point P(2, -3, 1) to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1} = \lambda \quad \dots(\text{Say})$$

The coordinates of any point on the line are given by

$$x = -1 + 2\lambda, y = 3 + 3\lambda, z = -1 - \lambda$$

Let the coordinates of M be

$$(-1 + 2\lambda, 3 + 3\lambda, -1 - \lambda) \quad \dots(1)$$

The direction ratios of PM are

$$-1 + 2\lambda - 2, 3 + 3\lambda + 3, -1 - \lambda - 1$$

$$\text{i.e. } 2\lambda - 3, 3\lambda + 6, -\lambda - 2$$

The direction ratios of the given line are 2, 3, -1.
 Since PM is perpendicular to the given line, we get

$$2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$$

$$\therefore 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$$

$$\therefore 14\lambda + 14 = 0$$

$$\therefore \lambda = -1.$$

Put $\lambda = -1$ in (1), the coordinates of M are

$(-1 - 2, 3 - 3, -1 + 1)$ i.e. $(-3, 0, 0)$.

\therefore length of perpendicular from P to the given line
 = PM

$$= \sqrt{(-3 - 2)^2 + (0 + 3)^2 + (0 - 1)^2}$$

$$= \sqrt{(25 + 9 + 1)}$$

$$= \sqrt{35} \text{ units.}$$

Solution2:

We know that the perpendicular distance from the point

$P|\vec{a}|$ to the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is given by

$$\sqrt{|\vec{a} - \vec{a}|^2 - \left[\frac{(\vec{a} - \vec{a}) \cdot \vec{b}}{|\vec{b}|} \right]^2} \quad \dots(1)$$

Here, $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{a} = -\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\therefore \vec{a} - \vec{a} = (2\hat{i} - 3\hat{j} + \hat{k}) - (-\hat{i} + 3\hat{j} - \hat{k})$$

$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\therefore |\vec{a} - \vec{a}|^2 = 3^2 + (-6)^2 + 2^2 = 9 + 36 + 4 = 49$$

$$\text{Also, } (\vec{a} - \vec{a}) \cdot \vec{b} = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$$

$$= (3)(2) + (-6)(3) + (2)(-1)$$

$$= 6 - 18 - 2$$

$$= -14$$

$$|\vec{b}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$$

Substituting these values in (1), we get

length of perpendicular from P to given line

= PM

$$= \sqrt{49 - \left(\frac{-14}{\sqrt{14}}\right)^2}$$

$$= \sqrt{49 - 14}$$

$$= \sqrt{35} \text{ units.}$$

Exercise 6.2 | Q 2 | Page 207

Find the co-ordinates of the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

Solution:

Let M be the foot of perpendicular drawn from the point P $(2\hat{i} - \hat{j} + 5\hat{k})$ on the line

$$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k}).$$

Let the position vector of the point M be

$$(11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$$

$$= (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}.$$

Then \overline{PM} = Position vector of M – Position vector of P

$$= [(11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}] - (2\hat{i} - \hat{j} + 5\hat{k})$$

$$= (9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}$$

Since PM is perpendicular to the given line which is parallel to

$$\vec{b} = 10\hat{i} - 4\hat{j} - 11\hat{k},$$

$$\overline{PM} \perp \vec{b}$$

$$\therefore \overline{PM} \cdot \vec{b} = 0$$

$$\therefore [(9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} - 11(-13 - 11\lambda)\hat{k}] \cdot (10\hat{i} - 4\hat{j} - 11\hat{k}) = 0$$

$$\therefore 10(9 + 10\lambda) - 4(-1 - 4\lambda) - 11(13 - 11\lambda) = 0$$

$$\therefore 90 + 100\lambda + 4 + 16\lambda + 143 + 121\lambda = 0$$

$$\therefore 237\lambda + 237 = 0$$

$$\therefore \lambda = -1$$

Putting this value of λ , we get the position vector of M as $\hat{i} + 2\hat{j} + 3\hat{k}$.

\therefore coordinates of the foot of perpendicular M are (1, 2, 3).

$$\text{Now, } \overline{PM} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 5\hat{k})$$

$$= -\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\therefore |\overline{PM}| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1 + 9 + 4}$$

$$= \sqrt{14}$$

Hence, the coordinates of the foot of perpendicular are (1, 2, 3) and length of perpendicular = $\sqrt{14}$ units.

Exercise 6.2 | Q 3 | Page 207

Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and } \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$$

Solution:

We know that the shortest distance between the skew lines

$$\vec{r} = \vec{a}_1 + \lambda\vec{b} \text{ and } \vec{r} = \vec{a}_2 + \mu\vec{b}_2 \text{ is given by } d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}.$$

$$\text{Here, } \vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k},$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b}_2 = \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\therefore \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

$$= (-10 + 12)\hat{i} - (-5 + 3)\hat{j} + (4 - 2)\hat{k}$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

and

$$\vec{a}_2 - \vec{a}_1 = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j})$$

$$= -3\hat{i} + 2\hat{k}$$

$$\therefore (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-3\hat{i} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= -3(2) + 0(2) + 2(2)$$

$$= -6 + 0 + 4$$

$$= -2$$

and

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{2^2 + 2^2 + 2^2}$$

$$= \sqrt{4 + 4 + 4}$$

$$= 2\sqrt{3}$$

∴ required shortest distance between the given lines

$$= \left| \frac{-2}{2\sqrt{3}} \right|$$

$$= \frac{1}{\sqrt{3}} \text{ units.}$$

Exercise 6.2 | Q 4 | Page 207

Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Solution:

The shortest distance between the lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2} \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}}$$

The equation of the given lines are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

$$\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$$

$$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6)$$

$$= -16 - 36 - 64$$

$$= -116$$

and

$$\begin{aligned} & (m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2 \\ &= (-6 + 2)^2 + (1 - 7)^2 + (1 - 7)^2 + (-14 + 6) \\ &= 16 + 36 + 64 \\ &= 116 \end{aligned}$$

Hence, the required shortest distance between the given lines

$$\begin{aligned} &= \left| \frac{-116}{\sqrt{116}} \right| \\ &= \sqrt{116} \\ &= 2\sqrt{29} \text{ units.} \end{aligned}$$

Exercise 6.2 | Q 5 | Page 207

Find the perpendicular distance of the point (1, 0, 0) from the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

Also find the co-ordinates of the foot of the perpendicular.

Solution:

Let PM be the perpendicular drawn from the point (1, 0, 0) to the line

$$\frac{x+1}{2} = \frac{y-3}{-3} = \frac{z+10}{8} = \lambda \quad \dots(\text{Say})$$

The coordinates of any point on the line are given by $x = -1 + 2\lambda$, $y = 3 + 3\lambda$, $z = 8 - \lambda$

Let the coordinates of M be

$$(-1 + 2\lambda, 3 + 3\lambda, -1 - \lambda) \quad \dots(1)$$

The direction ratios of PM are

$$-1 + 2\lambda - 2, 3 + 3\lambda + 3, -1 - \lambda - 1$$

$$\text{i.e. } 2\lambda - 3, 3\lambda + 6, -\lambda - 2$$

The direction ratios of the given line are 2, 3, 8.

Since PM is perpendicular to the given line, we get

$$2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$$

$$\therefore 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$$

$$\therefore 14\lambda + 14 = 0$$

$$\therefore \lambda = -1.$$

Put $\lambda = -1$ in (1), the coordinates of M are

$$(-1 - 2, 3 - 3, -1 + 1) \text{ i.e. } (-3, 0, 0).$$

\therefore length of perpendicular from P to the given line

$$= PM$$

$$= \sqrt{(-3 - 2)^2 + (0 + 3)^2 + (0 - 1)^2}$$

$$= \sqrt{25 + 9 + 1}$$

$$= \sqrt{35} \text{ units.}$$

Alternative Method :

We know that the perpendicular distance from the point

P(\vec{oo}) to the line $\vec{r} = \vec{a} + \lambda \vec{b}$ is given by

$$\sqrt{|\vec{oo} - \vec{a}|^2 - \left[\frac{(\vec{oo} - \vec{a}) \cdot \vec{b}}{|\vec{b}|} \right]^2} \quad \dots(1)$$

$$\text{Here, } \vec{oo} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{a} = -\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\therefore \vec{oo} - \vec{a} = (2\hat{i} - 3\hat{j} + \hat{k}) - (-\hat{i} + 3\hat{j} - \hat{k})$$

$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\therefore |\vec{oo} - \vec{a}|^2 = 3^2 + (-6)^2 + 2^2 = 9 + 36 + 4 = 49$$

$$\text{Also, } (\vec{oo} - \vec{a}) \cdot \vec{b} = (3\hat{i} - 6\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$$

$$= (3)(2) + (-6)(3) + (2)(-1)$$

$$= 6 - 18 - 2$$

$$= -14$$

$$|\vec{b}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$$

Substituting these values in (1), we get

length of perpendicular from P to given line

$$= PM$$

$$\begin{aligned}
 &= \sqrt{49 - \left(-\frac{14}{\sqrt{14}}\right)^2} \\
 &= \sqrt{49 - 14} \\
 &= \sqrt{35} \text{ units}
 \end{aligned}$$

or

$$2\sqrt{6} \text{ units}, (3, -4, 2).$$

Exercise 6.2 | Q 6 | Page 207

A(1, 0, 4), B(0, -11, 13), C(2, -3, 1) are three points and D is the foot of the perpendicular from A to BC. Find the co-ordinates of D.

Solution: Equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

\therefore the equation of the line BC passing through the points B (0, -11, 13) and C(2, -3, 1) is

$$\frac{x - 0}{2 - 0} = \frac{y + 11}{-3 + 11} = \frac{z - 13}{1 - 13}$$

$$\text{i.e. } x(2) = \frac{y + 11}{8} = \frac{z - 13}{-12} = \lambda \quad \text{..(Say)}$$

AD is the perpendicular from the point A(1, 0, 4) to the line BC.

The coordinates of any point on the line BC are given by

$$x = 2\lambda, y = -11 + 8\lambda, z = 13 - 12\lambda$$

Let the coordinates of D be $(2\lambda, -11 + 8\lambda, 13 - 12\lambda) \dots(1)$

\therefore the direction ratio of AD are

$$2\lambda - 1, -11 + 8\lambda - 0, 13 - 12\lambda - 4$$

$$\text{i.e. } 2\lambda - 1, -11 + 8\lambda, 9 - 12\lambda$$

The direction ratios of the line BC are 2, 8, -12.

Since AD is perpendicular to BC, we get

$$2(2\lambda - 1) + 8(-11 + 8\lambda) - 12(9 - 12\lambda) = 0$$

$$\therefore 4\lambda - 2 - 88 + 64\lambda - 108 + 144\lambda = 0$$

$$\therefore 212\lambda - 198 = 0$$

$$\therefore \lambda = \frac{198}{212} = \frac{99}{106}$$

Putting $\lambda = \frac{99}{106}$ in (1), the coordinates of D are

$$\left(\frac{198}{106}, -11 + \frac{792}{106}, 13 - \frac{1188}{106} \right)$$

$$\text{i.e. } \left(\frac{198}{106}, \frac{-374}{106}, \frac{190}{106} \right),$$

$$\text{i.e. } \left(\frac{99}{53}, \frac{-187}{53}, \frac{95}{53} \right).$$

Exercise 6.2 | Q 7.1 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$$\bar{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k}) \text{ and } \bar{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

Solution: The shortest distance between the lines

$$\bar{r} = \bar{a}_1 + \lambda\bar{b}_1 \text{ and } \bar{r} = \bar{a}_2 + \mu\bar{b}_2 \text{ is given by}$$

$$d = \left| \frac{(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)}{|\bar{b}_1 \times \bar{b}_2|} \right|.$$

$$\text{Here, } \bar{a}_1 = \hat{i} - \hat{j}, \bar{a}_2 = 2\hat{i} - \hat{j}, \bar{b}_1 = 2\hat{i} + \hat{k}, \bar{b}_2 = \hat{i} + \hat{j} - \hat{k}.$$

$$\therefore \bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= (0 - 1)\hat{i} - (-2 - 1)\hat{j} + (2 - 0)\hat{k}$$

$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

And

$$\bar{a}_2 - \bar{a}_1 = (2\hat{i} - \hat{j}) - (\hat{i} - \hat{j} - \hat{i})$$

$$\therefore (\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2) = \hat{i} \cdot (-\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 1(-1) + 0(3) + 0(2)$$

$$= -1$$

and

$$|\bar{b}_1 \times \bar{b}_2| = \sqrt{(-1)^2 + 3^2 + 2^2}$$

$$= \sqrt{1 + 9 + 4}$$

$$= \sqrt{14}$$

\therefore the shortest distance between the given lines

$$= \left| \frac{-1}{\sqrt{14}} \right|$$

$$= \frac{1}{\sqrt{14}} \text{ unit}$$

Hence, the given line do not intersect.

Exercise 6.2 | Q 7.2 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

Solution:

The shortest distance between the lines

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}}$$

The equation of the given lines are

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

$$\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$$

$$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6)$$

$$= -16 - 36 - 64$$

$$= -116$$

and

$$(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2$$

$$= (-6 + 2)^2 + (1 - 7)^2 + (1 - 7)^2 + (-14 + 6)$$

$$= 16 + 36 + 64$$

$$= 116$$

Hence, the required shortest distance between the given lines

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \sqrt{116}$$

$$= 2\sqrt{29} \text{ units}$$

or

The shortest distance between the lines

$$= \frac{282}{\sqrt{3830}} \text{ units}$$

Hence, the given lines do not intersect.

Exercise 6.2 | Q 8 | Page 207

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find k.

Solution:

The lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

intersect, if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

The equations of the given lines are

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \text{ and } \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

$$\therefore x_1 = 1, y_1 = -1, z_1 = 1, x_2 = 3, y_2 = k, z_2 = 0,$$

$$l_1 = 2, m_1 = 3, n_1 = 4, l_2 = 1, m_2 = 2, n_2 = 1.$$

Since these lines intersect, we get

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore 2(3-8) - (k+1)(2-4) - 1(4-3) = 0$$

$$\therefore -10 + 2(k+1) - 1 = 0$$

$$\therefore 2(k+1) = 11$$

$$\therefore k+1 = 11/2$$

$$\therefore k = 9/2.$$

EXERCISE 6.3 [PAGE 216]

Exercise 6.3 | Q 1 | Page 216

Find the vector equation of a plane which is at 42 unit distance from the origin and which is normal to the vector $2\hat{i} + \hat{j} - 2\hat{k}$

Solution:

If \hat{n} is a unit vector along the normal and p is the length of the perpendicular from origin to the plane, then the vector equation of the plane $\vec{r} \cdot \hat{n} = p$

Here, $\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $p = 42$

$$\therefore |\vec{n}| = \sqrt{2^2 + 1^2 + (-2)^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k})$$

\therefore the vector equation of the required plane is

$$\vec{r} \cdot \left[\frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k}) \right] = 42$$

$$\text{i.e. } \vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 126.$$

Exercise 6.3 | Q 2 | Page 216

Find the perpendicular distance of the origin from the plane $6x - 2y + 3z - 7 = 0$.

Solution: The equation of the plane is

$$6x - 2y + 3z - 7 = 0$$

\therefore its vector equation is

$$\vec{r} \cdot (6\hat{i} - 2\hat{j} + 3\hat{k}) = 7 \quad \dots(1)$$

$$\text{where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\therefore \vec{n} = 6\hat{i} - 2\hat{j} + 3\hat{k}$ is normal to the plane.

$$|\vec{n}| = \sqrt{6^2 + (-2)^2 + 3^2}$$

$$= \sqrt{49}$$

$$= 7$$

Unit vector along \vec{n} is

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{6\hat{i} - 2\hat{j} + 3\hat{k}}{7}$$

Dividing bothsides of (1) by 7, we get

$$\vec{r} \cdot \left(\frac{6\hat{i} - 2\hat{j} + 3\hat{k}}{7} \right) = \frac{7}{7}$$

$$\therefore \vec{r} \cdot \hat{n} = 1$$

Comparing with normal form of equation of the plane $\vec{r} \cdot \hat{n} = p$ it follows that length of perpendicular from origin is 1 unit.

Alternative Method :

The equation of the plane is $6x - 2y + 3z - 7 = 0$

$$\text{i.e.} \left(\frac{6}{6^2 + (-2)^2 + 3^2} \right)x - \left(\frac{2}{\sqrt{6^2 + (-2)^2 + 3^2}} \right)y + \left(\frac{3}{\sqrt{6^2 + (-2)^2 + 3^2}} \right)z = \frac{7}{\sqrt{6^2 + (-2)^2 + 3^2}}$$

$$\text{i.e.} \frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z = \frac{7}{7} = 1$$

This is the normal form of the equation of plane.

\therefore perpendicular distance of the origin frm the plane is $p = 1$ unit.

Exercise 6.3 | Q 3 | Page 216

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane

$$2x + 6y - 3z = 63.$$

Solution:

The equation of the plane is $2x + 6y - 3z = 63$.

Dividing each term by

$$\sqrt{2^2 + 6^2 + (-3)^2}$$

$$= \sqrt{49}$$

$$= 7,$$

we get

$$\frac{2}{7}x + \frac{6}{7}y - \frac{3}{7}z = \frac{63}{7} = 9$$

This is the normal form of the equation of plane.

∴ the direction cosines of the perpendicular drawn from the origin to the plane are

$$l = \frac{2}{7}, m = \frac{6}{7}, n = -\frac{3}{7}$$

and length of perpendicular from origin to the plane is $p = 9$.

∴ the coordinates of the foot of the perpendicular from the origin to the plane are

$$(lp, mp, np) \text{ i.e. } \left(\frac{18}{7}, \frac{54}{7}, -\frac{27}{7} \right)$$

Exercise 6.3 | Q 4 | Page 216

Reduce the equation $\vec{r} \cdot (3\hat{i} + 4\hat{j} + 12\hat{k}) = 78$ to normal form and hence find

- (i) the length of the perpendicular from the origin to the plane
- (ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\vec{r} \cdot \hat{n} = p$ where \hat{n} is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

$$\text{Given plane is } \vec{r} \cdot (3\hat{i} + 4\hat{j} + 12\hat{k}) = 78 \quad \dots(1)$$

$\vec{n} = 3\hat{i} + 4\hat{j} + 12\hat{k}$ is normal to the plane

$$\therefore |\vec{n}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$$

Dividing both sides of (1) by 13, get

$$\vec{r} \cdot \left(\frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} \right) = \frac{78}{13}$$

$$\text{i.e. } \vec{r} \cdot \left(\frac{3}{13}\hat{i} + \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k} \right) = 6$$

This is the normal form of the equation of plane.

Comparing with $\vec{r} \cdot \hat{n} = p$,

(i) the length of the perpendicular from the origin to plane is 6.

(ii) direction cosines of the normal are $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$.

Exercise 6.3 | Q 5 | Page 216

Find the vector equation of the plane passing through the point having position vector $\hat{i} + \hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 5\hat{j} + 6\hat{k}$.

Solution:

The vector equation of the plane passing through the point $A(\vec{a})$ and perpendicular to the vector \vec{n} is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

Here,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k},$$

$$\vec{n} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\therefore \vec{a} \cdot \vec{n} = (\hat{i} + \hat{j} + \hat{k}) \cdot (4\hat{i} + 5\hat{j} + 6\hat{k})$$

$$= (1)(4) + (1)(5) + (1)(6)$$

$$= 4 + 5 + 6$$

$$= 15$$

\therefore the vector equation of the required plane is

$$\vec{r} \cdot (4\hat{i} + 5\hat{j} + 6\hat{k}) = 15.$$

Exercise 6.3 | Q 6 | Page 216

Find the Cartesian equation of the plane passing through $A(-1, 2, 3)$, the direction ratios of whose normal are 0, 2, 5.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction ratios of whose normal are a, b, c , is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

\therefore the cartesian equation of the required plane is

$$0(x + 1) + 2(y - 2) + 5(z - 3) = 0$$

$$\text{i.e. } 0 + 2y - 4 + 5z - 15 = 0$$

$$\text{i.e. } 2y + 5z = 19.$$

Exercise 6.3 | Q 7 | Page 216

Find the Cartesian equation of the plane passing through $A(7, 8, 6)$ and parallel to the XY plane.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction ratios of whose normal are a, b, c , is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

The required plane is parallel to XY-plane.

\therefore it is perpendicular to Z-axis i.e. Z-axis is normal to the plane. Z-axis has direction ratios 0, 0, 1.

The plane passes through (7, 8, 6).

\therefore the cartesian equation of the required plane is

$$0(x - 7) + 0(y - 8) + 1(z - 6) = 0$$

i.e. $z = 6$.

Exercise 6.3 | Q 8 | Page 216

The foot of the perpendicular drawn from the origin to a plane is $M(1,0,0)$. Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through $A(\vec{a})$ and perpendicular to \vec{n} is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

$M(1,0,0)$ is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.

If \vec{m} is the position vector of M, then $\vec{m} = \hat{i}$.

Normal to the plane is

$$\vec{n} = \overrightarrow{OM} = \hat{i}$$

$$\vec{m} \cdot \vec{n} = \hat{i} \cdot \hat{i} = 1$$

\therefore the vector equation of the required plane is

$$\vec{r} \cdot \hat{i} = 1.$$

Exercise 6.3 | Q 9 | Page 216

Find the vector equation of the plane passing through the point $A(-2, 7, 5)$ and parallel to vector $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors \bar{b} and \bar{c} is

$$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c}) \quad \dots(1)$$

$$\text{Here, } \bar{a} = -2\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\bar{b} = 4\hat{i} - \hat{j} + 3\hat{k},$$

$$\bar{c} = \hat{i} + \hat{j} + \hat{k}$$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (-1 - 3)\hat{i} - (4 - 3)\hat{j} + (4 + 1)\hat{k}$$

$$= -4\hat{i} - \hat{j} + 5\hat{k}$$

$$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (-2\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (-4\hat{i} - \hat{j} + 5\hat{k})$$

$$= (-2)(-4) + (7)(-1) + (5)(5)$$

$$= 8 - 7 + 2$$

$$= 26$$

$$\therefore \text{From (1), the vector equation of the required plane is } \bar{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26.$$

Exercise 6.3 | Q 10 | Page 216

Find the cartesian equation of the plane

$$\bar{r} = (5\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k}).$$

Solution:

The equation $\bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$ represents a plane passing through a point having position vector \bar{a} and parallel to vectors \bar{b} and \bar{c} .

Here,

$$\bar{a} = 5\hat{i} - 2\hat{j} - 3\hat{k},$$

$$\bar{b} = \hat{i} + \hat{j} + \hat{k},$$

$$\bar{c} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= (3 + 2)\hat{i} - (3 - 1)\hat{j} + (-2 - 1)\hat{k}$$

$$= 5\hat{i} - 2\hat{j} - 3\hat{k}$$

$$= \bar{a}$$

Also,

$$\bar{a} \cdot (\bar{b} \times \bar{c})$$

$$= \bar{a} \cdot \bar{a} = |\bar{a}|^2$$

$$= (5)^2 + (-2)^2 + (3)^2$$

$$= 38$$

The vector equation of the plane passing through A(\bar{a}) and parallel to \bar{b} and \bar{c} is

$$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c})$$

\therefore the vector equation of the given plane is

$$\bar{r} \cdot (5\hat{i} - 2\hat{j} - 3\hat{k}) = 38$$

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then this equation becomes

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (5\hat{i} - 2\hat{j} - 3\hat{k}) = 38$$

$$\therefore 5x - 2y - 3z = 38.$$

This is the cartesian equation of the required plane.

Exercise 6.3 | Q 11 | Page 216

Find the vector equation of the plane which makes intercepts 1, 1, 1 on the co-ordinates axes.

Solution:

The vector equation of the plane passing through $A(\bar{a}), B(\bar{b}), C(\bar{c})$, where A, B, C are non-collinear is $\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC}) \dots(1)$

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

\therefore it passes through the three non-collinear points $A = (1, 0, 0)$, $B = (0, 1, 0)$, $C = (0, 0, 1)$

$$\therefore \bar{a} = \hat{i}, \bar{b} = \hat{j}, \bar{c} = \hat{k}$$

$$\overline{AB} = \bar{b} - \bar{a} = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$$

$$\therefore \overline{AC} = \bar{c} - \bar{a} = \hat{k} - \hat{i} = -\hat{i} + \hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= (1 - 0)\hat{i} - (-1 - 0)\hat{j} + (0 + 1)\hat{k}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

Also,

$$\bar{a} \cdot (\overline{AB} \times \overline{AC})$$

$$= \hat{i} \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$= 1 \times 1 + 0 \times 1 + 0 \times 1$$

$$= 1$$

\therefore from(1) the vector equation of the required plane is $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$.

EXERCISE 6.4 [PAGE 220]

Exercise 6.4 | Q 1 | Page 220

Find the angle between planes $\bar{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 13$ and $\bar{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 31$.

Solution:

Find the angle between planes $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 13$ and $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 31$.

The acute angle θ between the planes

$\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is given by

$$\cos \theta = \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right| \quad \dots(1)$$

Here,

$$\vec{n}_1 = \hat{i} + \hat{j} + 2\hat{k},$$

$$\vec{n}_2 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore \vec{n}_1 \cdot \vec{n}_2$$

$$= (\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k})$$

$$= (1)(2) + (1)(-1) + (2)(1)$$

$$= 2 - 1 + 2$$

$$= 3$$

Also,

$$|\vec{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$|\vec{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

\therefore from (1), we have

$$\cos \theta = \left| \frac{3}{\sqrt{6}\sqrt{6}} \right|$$

$$= \frac{3}{6}$$

$$= \frac{1}{2} \cos 60^\circ$$

$$\therefore \theta = 60^\circ.$$

Exercise 6.4 | Q 2 | Page 220

Find the acute angle between the line

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k}) \text{ and the plane } \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 0.$$

Solution:

The acute angle θ between the line $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r} \cdot \vec{n} = d$ is given by

$$\sin \theta = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{b}| |\vec{n}|} \right| \quad \dots(1)$$

$$\text{Here, } \vec{b} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{n} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore \vec{b} \cdot \vec{n} = (2\hat{i} + 3\hat{j} - 6\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})$$

$$= (2)(2) + (3)(-1) + (-6)(1)$$

$$= 4 - 3 - 6$$

$$= -5$$

$$\text{Also, } |\vec{b}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7$$

$$|\vec{n}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

\therefore from (1), we have

$$\sin \theta = \left| \frac{-5}{7\sqrt{6}} \right| = \frac{5}{7\sqrt{6}}$$

$$\therefore \theta = \sin^{-1} \left(\frac{5}{7\sqrt{6}} \right).$$

Exercise 6.4 | Q 3 | Page 220

Show that the line $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ are coplanar. Find the equation of the plane determined by them.

Solution:

The lines $\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2$ are coplanar if $\vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2) = \vec{a}_2 \cdot (\vec{b}_1 \times \vec{b}_2)$

$$\text{Here } \vec{a}_1 = 2\hat{j} - 3\hat{k}, \vec{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = (2\hat{i} + 6\hat{j} + 3\hat{k}) - (2\hat{j} - 3\hat{k})$$

$$= 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= (8 - 9)\hat{i} - (4 - 6)\hat{j} + (3 - 4)\hat{k}$$

$$= -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})$$

$$= 0(-1) + 2(2) + (-3)(-1)$$

$$= 0 + 4 + 3$$

$$= 7$$

$$\therefore \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2) = \vec{a}_2 \cdot (\vec{b}_1 \times \vec{b}_2)$$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$$\therefore \vec{r} \cdot (\vec{b}_1 \times \vec{b}_2) = \vec{a}_1 \cdot (\vec{b}_1 \times \vec{b}_2)$$

$$\text{i.e. } \vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$$

Hence, the given lines are coplanar and the equation of the plane determined by these lines is

$$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7.$$

Exercise 6.4 | Q 4 | Page 220

Find the distance of the point $4\hat{i} - 3\hat{j} + \hat{k}$ from the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 21$.

Solution:

The distance of the point $A(\bar{a})$ from the plane $\bar{r} \cdot \bar{n} = p$ is given by $d = \frac{|\bar{a} \cdot \bar{n} - p|}{|\bar{n}|}$... (1)

Here, $\bar{a} = 4\hat{i} - 3\hat{j} + \hat{k}$, $\bar{n} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $p = 21$

$$\therefore \bar{a} \cdot \bar{n} = (4\hat{i} - 3\hat{j} + \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 6\hat{k})$$

$$= (4)(2) + (-3)(3) + (1)(-6)$$

$$= 8 - 9 - 6$$

$$= -7$$

$$\text{Also, } |\bar{n}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7$$

\therefore from (1), the required distance

$$= \frac{|-7 - 21|}{7}$$

$$= 4 \text{ units.}$$

Exercise 6.4 | Q 5 | Page 220

Find the distance of the point $(1, 1, -1)$ from the plane $3x + 4y - 12z + 20 = 0$.

Solution:

The distance of the point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is

$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

\therefore the distance of the point $(1, 1, -1)$ from the plane $3x + 4y - 12z + 20 = 0$ is

$$\left| \frac{3(1) + 4(1 - 12(-1) + 20)}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$$

$$= \left| \frac{3 + 4 + 12 + 20}{\sqrt{9 + 16 + 144}} \right|$$

$$= \frac{39}{\sqrt{169}}$$

$$= \frac{39}{13}$$

$$= 3 \text{ units.}$$

MISCELLANEOUS EXERCISE 6 A [PAGES 207 - 209]

Miscellaneous Exercise 6 A | Q 1 | Page 207

Find the vector equation of the line passing through the point having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and parallel to $6\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the line passing through $A(\bar{a})$ and parallel to the vector \bar{b} is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

\therefore the vector equation of the line passing through the point having position vector

$3\hat{i} + 4\hat{j} - 7\hat{k}$ and parallel to the vector $6\hat{i} - \hat{j} + \hat{k}$ is

$$\bar{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(6\hat{i} - \hat{j} + \hat{k}).$$

Miscellaneous Exercise 6 A | Q 2 | Page 207

Find the vector equation of the line which passes through the point $(3, 2, 1)$ and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

Solution:

The vector equation of the line passing through $A(\bar{a})$ and parallel to the vector \bar{b} is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

\therefore the vector equation of the line passing through the point having position vector $3\hat{i} + 2\hat{j} + \hat{k}$ and parallel to the vector

$$2\hat{i} + 2\hat{j} - 3\hat{k} \text{ is } \bar{r} = (3\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k}).$$

Miscellaneous Exercise 6 A | Q 3 | Page 208

Find the Cartesian equations of the line which passes through the point $(-2, 4, -5)$ and parallel to the line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{z+5}{6}$.

Solution:

The line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{z+5}{6}$ has direction ratios 3, 5, 6. The required line has direction ratios 3, 5, 6 as it is parallel to the given line.

It passes through the point $(-2, 4, -5)$.

The cartesian equation of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

\therefore the required cartesian equation of the line are

$$\frac{x - (-2)}{3} = \frac{y - 4}{5} = \frac{z - (-5)}{6}$$

$$\text{i.e. } \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}.$$

Miscellaneous Exercise 6 A | Q 4 | Page 208

Obtain the vector equation of the line $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

Solution:

The cartesian equations of the line are $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

This line is passing through the point $A(-5, -4, -5)$ and having direction ratios 3, 5, 6.

Let \vec{a} be the position vector of the point A w.r.t. the origin and \vec{b} be the vector parallel to the line.

Then $\bar{a} = -5\hat{i} - 4\hat{j} - 5\hat{k}$ and $\bar{b} = 3\hat{i} + 5\hat{j} + 6\hat{k}$.

The vector equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.

\therefore the vector equation of the required line is

$$\bar{r} = (-5\hat{i} - 4\hat{j} - 6\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k}).$$

Miscellaneous Exercise 6 A | Q 5 | Page 208

Find the vector equation of the line which passes through the origin and the point (5, -2, 3).

Solution:

Let \bar{b} be the position vector of the point B(5, -2, 3).

$$\text{Then } \bar{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$$

Origin has position vector $\bar{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

The vector equation the line passing through $A(\bar{a})$ and $B(\bar{b})$ is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$ where λ is a scalar.

\therefore the vector equation of the required line is

$$\bar{r} = \bar{0} + \lambda(\bar{b} - \bar{0}) = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k}).$$

Miscellaneous Exercise 6 A | Q 6 | Page 208

Find the Cartesian equations of the line which passes through points (3, -2, -5) and (3, -2, 6).

Solution:

Let A = (3, -2, -5) and (3, -2, 6)

The direction ratios of the line AB are

$3 - 3, -2 - (-2), 6 - (-5)$ i.e. 0, 0, 11.

The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$x = x_1 + a\lambda, y = y_1 + b\lambda, z = z_1 + c\lambda$$

\therefore The parametric equations of the line passing through (3, -2, -5) and having direction ratios are 0, 0, 11 are

$$x = 3 + (0)\lambda, y = -2 + 0(\lambda), z = -5 + 11\lambda$$

$$\text{i.e. } x = 3, y = -2, z = 11\lambda - 5$$

\therefore the cartesian equations of the line are

$$x = 3, y = -2, z = 11\lambda - 5, \lambda \text{ is a scalar.}$$

Miscellaneous Exercise 6 A | Q 7 | Page 208

Find the Cartesian equations of the line passing through A(3, 2, 1) and B(1, 3, 1).

Solution: The direction ratios of the line AB are $3 - 1, 2 - 3, 1 - 1$ i.e. 2, -1, 0.

The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$x = x_1 + a\lambda, y = y_1 + b\lambda, z = z_1 + c\lambda$$

\therefore the parametric equations of the line passing through (3, 2, 1) and having direction ratios 2, -1, 0 are

$$x = 3 + 2\lambda, y = 2 - \lambda, z = 1 + 0(\lambda)$$

$$\therefore x - 3 = 2\lambda, y - 2 = -\lambda, z = 1$$

$$\therefore \frac{x - 3}{2} = \frac{y - 2}{-1} = \lambda, z = 1$$

\therefore the cartesian equations of the required line are

$$\frac{x - 3}{2} = \frac{y - 2}{-1}, z = 1.$$

Miscellaneous Exercise 6 A | Q 8 | Page 208

Find the Cartesian equations of the line passing through the point

A(1, 1, 2) and perpendicular to the vectors

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}$$

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}.$$

\therefore it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, -1.

$$\therefore p + 2q + r = 0, 3 + 2q - r = 0$$

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1}$$

$$\therefore \frac{p}{-1} = \frac{q}{1} = \frac{r}{-1}$$

\therefore the required line has direction ratios -1, 1, -1.

The cartesian equations of the line passing through (x_1, y_1, z_1) and

$$\text{having direction ratios } a, b, c \text{ are } \frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

\therefore the cartesian equation of the line passing through the point (1, 1, 2) and having directions ratios -1, 1, -1 are

$$\frac{x - 1}{-1} = \frac{y - 1}{1} = \frac{z - 2}{-1}.$$

Miscellaneous Exercise 6 A | Q 9 | Page 208

Find the Cartesian equations of the line which passes through the point (2, 1, 3) and perpendicular to the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \text{ and } \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k} \text{ and } \vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}.$$

\therefore it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, -1.

$$\therefore p + 2q + r = 0, 3 + 2q - r = 0$$

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1}$$

$$\therefore \frac{p}{2} = \frac{q}{-7} = \frac{r}{4}$$

\therefore the required line has direction ratios 2, -7, 4.

The cartesian equations of the line passing through (x_1, y_1, z_1) and

$$\text{having direction ratios } a, b, c \text{ are } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

\therefore the cartesian equation of the line passing through the point (2, -7, 4) and having directions ratios 2, -7, 4 are

$$\frac{x-2}{2} = \frac{y+7}{-7} = \frac{z-4}{4}.$$



Miscellaneous Exercise 6 A | Q 10 | Page 208

Find the vector equation of the line which passes through the origin and intersect the line $x - 1 = y - 2 = z - 3$ at right angle.

Solution:

$$\text{The given line is } \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1} = \lambda \quad \dots(\text{Say})$$

$$\therefore \text{ coordinates of any point on the line are } x = \lambda + 1, y = \lambda + 2, z = \lambda + 3$$

$$\therefore \text{ position vector of any point on the line is } (\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (\lambda + 3)\hat{k} \quad \dots(1)$$

If \vec{b} is parallel to the given line whose direction ratios are 1, 1, 1 then $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

Let the required line passing through O meet the given line at M.

$$\therefore \text{ position vector of M} \quad +$$

$$= \vec{m} = (\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (\lambda + 3)\hat{k} \quad \dots[\text{By (1)}]$$

The required line is perpendicular to given line

$$\therefore \vec{OM} \cdot \vec{b} = 0$$

$$\therefore [(\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (\lambda + 3)\hat{k}] \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\therefore (\lambda + 1) \times 1 + (\lambda + 2) \times 1 + (\lambda + 3) \times 1 = 0$$

$$\therefore 3\lambda + 6 = 0$$

$$\therefore \lambda = -2$$

$$\therefore \vec{m} = (-2 + 1)\hat{i} + (-2 + 2)\hat{j} + (-2 + 3)\hat{k} = -\hat{i} + \hat{k}$$

The vector equation of the line passing through A(\bar{a}) and B(\bar{b}) is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$, λ is a scalar.

\therefore the vector equation of the line passing through O($\bar{0}$) and M(\bar{m}) is $\bar{r} = \bar{0} + \lambda(\bar{m} - \bar{0}) = \lambda(-\hat{i} + \hat{k})$ where λ is a scalar.

Hence, vector equation of the required line is $\bar{r} = \lambda(-\hat{i} + \hat{k})$.

Miscellaneous Exercise 6 A | Q 11 | Page 208

Find the value of λ so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.

Solution:

The equations of the given lines are

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2} \dots(1)$$

and

$$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \dots(2)$$

Equation (1) can be written as:

$$\frac{-(x-1)}{3} = \frac{7(y-2)}{2\lambda} = \frac{z-3}{2}$$

$$\text{i.e. } \frac{x-1}{-3} = \frac{y-2}{\frac{2\lambda}{7}} = \frac{z-3}{2}$$

The direction ratios of this line are

$$a_1 = -3, b_1 = \frac{2\lambda}{7}, c_1 = 2$$

Equation (2) can be written as :

$$\frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\text{i.e. } \frac{x-1}{-\frac{3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

The direction ratios of this line are

$$a_2 = \frac{-3\lambda}{7}, b_2 = 1, c_2 = -5$$

Since the lines (1) and (2) are at right angles,

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\therefore (-3) \left(\frac{-3\lambda}{7} \right) + \left(\frac{2\lambda}{7} \right) (1) + 2(-5) = 0$$

$$\therefore \left(\frac{9\lambda}{7} \right) + \left(\frac{2\lambda}{7} \right) - 10 = 0$$

$$\therefore \frac{11\lambda}{7} = 10$$

$$\therefore \lambda = \frac{70}{11}.$$

Miscellaneous Exercise 6 A | Q 12 | Page 208

Find the acute angle between the lines

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2} \quad \text{and} \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}.$$

Solution:

Let \bar{a} and \bar{b} be the vectors in the direction of the lines

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2} \text{ and } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$$

respectively.

$$\text{Then } \bar{a} = \hat{i} - \hat{j} + 2\hat{k}, \bar{b} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\therefore \bar{a} \cdot \bar{b} = (\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k})$$

$$= (1)(2) + (-1)(1) + (2)(1)$$

$$= 2 - 1 + 2$$

$$= 3$$

$$|\bar{a}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$$

$$|\bar{b}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

If θ is the angle between the lines, then

$$\cos \theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| |\bar{b}|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \theta = 60^\circ.$$

Miscellaneous Exercise 6 A | Q 13 | Page 208

Find the acute angle between the lines $x = y, z = 0$ and $x = 0, z = 0$.

Solution:

The equations $x = y, z = 0$ can be written as $\frac{x}{1} = \frac{y}{1}, z = 0$.

\therefore the direction ratios of the line are 1, 1, 0.

The direction ratios of the line $x = 0, z = 0$, i.e., Y-axis are 0, 1, 0.

\therefore its direction ratios are 0, 1, 0.

Let \bar{a} and \bar{b} be the vectors in the direction of the lines $x = y, z = 0$ and $x = 0, z = 0$.

$$\text{Then } \bar{a} = \hat{i} + \hat{j}, \bar{b} = \hat{j}$$

$$\therefore \bar{a} \cdot \bar{b} = (\hat{i} + \hat{j}) \cdot \hat{j}$$

$$= (1)(0) + (1)(1) + (0)(0)$$

$$= 1$$

$$|\bar{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\bar{b}| = |\hat{j}| = 1$$

If θ is the acute angle between the lines, then

$$\cos \theta = \left| \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| |\bar{b}|} \right| = \left| \frac{1}{\sqrt{2} \times 1} \right| = \frac{1}{\sqrt{2}} = \cos 45^\circ.$$

$$\therefore \theta = 45^\circ.$$

Miscellaneous Exercise 6 A | Q 14 | Page 208

Find the acute angle between the lines $x = -y, z = 0$ and $x = 0, z = 0$.

Solution:

The equations $x = -y, z = 0$ can be written as $\frac{x}{1} = \frac{y}{1}, z = 0$.

\therefore the direction ratios of the line are 1, 1, 0.

The direction ratios of the line $x = 0, z = 0$, i.e., Y-axis are 0, 1, 0.

\therefore its direction ratios are 0, 1, 0.

Let \bar{a} and \bar{b} be the vectors in the direction of the lines $x = y, z = 0$ and $x = 0, z = 0$.

$$\text{Then } \bar{a} = \hat{i} + \hat{j}, \bar{b} = \hat{j}$$

$$\therefore \bar{a} \cdot \bar{b} = (\hat{i} + \hat{j}) \cdot \hat{j}$$

$$= (1)(0) + (1)(1) + (0)(0)$$

$$= 1$$

$$|\bar{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\bar{b}| = |\hat{j}| = 1$$

If θ is the acute angle between the lines, then

$$\cos \theta = \left| \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| |\bar{b}|} \right| = \left| \frac{1}{\sqrt{2} \times 1} \right| = \frac{1}{\sqrt{2}} = \cos 45^\circ.$$

$$\therefore \theta = 45^\circ.$$

Miscellaneous Exercise 6 A | Q 15 | Page 208

Find the co-ordinates of the foot of the perpendicular drawn from the point (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

Solution: Let $P = (0, 2, 3)$

Let M be the foot of the perpendicular drawn from P to the line



$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda \quad \text{..(Say)}$$

The coordinates of any point on the line are given by

$$x = 5\lambda - 3, y = 2\lambda + 1, z = 3\lambda - 4$$

$$\text{Let } M = (5\lambda - 3, 2\lambda + 1, 3\lambda - 4) \quad \dots(1)$$

The direction ratios of PM are

$$5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$$

$$\text{i.e. } 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$$

Since, PM is perpendicular to the line whose direction ratios are 5, 2, 3,

$$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$$

$$\therefore 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$$

$$\therefore 38\lambda - 38 = 0$$

$$\therefore \lambda = 1$$

Substituting $\lambda = 1$ in (1), we get

$$M = (5 - 3, 2 + 1, 3 - 4) = (2, 3, -1).$$

Hence, the coordinates of the foot of perpendicular are (2, 3, -1).

Miscellaneous Exercise 6 A | Q 16.1 | Page 208

By computing the shortest distance determine whether following lines intersect each other :

$$\bar{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{and} \quad \bar{r} = (2\hat{i} + 2\hat{j} - 3\hat{k}) + \mu(\hat{i} + \hat{j} - 2\hat{k})$$

Solution: The shortest distance between the lines

$$\bar{r} = \bar{a}_1 + \lambda\bar{b}_1 \quad \text{and} \quad \bar{r} = \bar{a}_2 + \mu\bar{b}_2 \quad \text{is given by}$$

$$d = \left| \frac{(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)}{|\bar{b}_1 \times \bar{b}_2|} \right|.$$

$$\text{Here, } \bar{a}_1 = \hat{i} + \hat{j} - \hat{k}, \bar{a}_2 = 2\hat{i} + 2\hat{j} - 3\hat{k},$$

$$\bar{b}_1 = 2\hat{i} - \hat{j} + \hat{k}, \bar{b}_2 = \hat{i} + \hat{j} - 2\hat{k}.$$

$$\begin{aligned}\therefore \bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2 &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} \\ &= (2 - 1)\hat{\mathbf{i}} - (-4 - 1)\hat{\mathbf{j}} + (4 + 1)\hat{\mathbf{k}} \\ &= \hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}}\end{aligned}$$

and

$$\begin{aligned}\bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1 &= (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \\ \therefore (\bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1) \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2) &= \hat{\mathbf{i}} \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \\ &= 1(-1) + 0(3) + 0(2) \\ &= -1\end{aligned}$$

and

$$\begin{aligned}|\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2| &= \sqrt{(-1)^2 + 3^2 + 2^2} \\ &= \sqrt{1 + 9 + 4} \\ &= \sqrt{14}\end{aligned}$$

Shortest distance between the lines is 0.

\therefore the lines intersect each other.

Miscellaneous Exercise 6 A | Q 16.2 | Page 208

By computing the shortest distance determine whether the following line intersect each other : $\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5}$ and $x-6 = y-8 = z+2$.

Solution: The shortest distance between the lines

$$\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5} \text{ and } x-6 = y-8 = z+2 \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}}$$

The equation of the given lines are

$$\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5} \text{ and } x-6 = y-8 = z+2$$

$$\therefore x_1 = 5, y_1 = 7, z_1 = 3, x_2 = 6, y_2 = 8, z_2 = 2,$$

$$l_1 = 4, m_1 = 5, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & 5 & 5 \\ -6 & -8 & 2 \end{vmatrix}$$

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6)$$

$$= -16 - 36 - 64$$

$$= -116$$

and

$$(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - l_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2$$

$$= (-6 + 2)^2 + (1 - 7)^2 + (1 - 7)^2 + (-14 + 6)$$

$$= 16 + 36 + 64$$

$$= 116$$

Hence, the required shortest distance between the given lines

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \sqrt{116}$$

$$= 2\sqrt{29} \text{ units}$$

or

Shortest distance between the lines is 0.

\therefore the lines intersect each other.

Miscellaneous Exercise 6 A | Q 17 | Page 208

If the lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \text{ and } \frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$$

intersect each other, find m.

Solution:

The lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$ intersect, if

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \quad \dots(1)$$

Here, $(x_1, y_1, z_1) \equiv (1, -1, 1)$,

$(x_2, y_2, z_2) \equiv (2, -m, 2)$,

$a_1 = 2, b_1 = 3, c_1 = 4$,

$a_2 = 1, b_2 = 2, c_2 = 1$

Substituting these values in (1), we get

$$\begin{vmatrix} 2-1 & -m+1 & 2-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & 1-m & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore 1(3-8) - (1-m)(2-4) + 1(4-3) = 0$$

$$\therefore -5 + 2 - 2m + 1 = 0$$

$$\therefore -2m = 2$$

$$\therefore m = -1.$$

Miscellaneous Exercise 6 A | Q 18 | Page 208

Find the vector and Cartesian equations of the line passing through the point $(-1, -1, 2)$ and parallel to the line $2x - 2 = 3y + 1 = 6z - 2$.

Solution:

Let \vec{a} be the position vector of the point $A(-1, -1, 2)$ w.r.t. the origin.

$$\text{Then } \vec{a} = -\hat{i} - \hat{j} + 2\hat{k}$$

The equation of given line is

$$2x - 2 = 3y + 1 = 6z - 2$$

$$\therefore 2(x - 1) = 3\left(y + \frac{1}{3}\right) = 6\left(z - \frac{1}{3}\right)$$

$$\therefore \frac{x-1}{\left(\frac{1}{2}\right)} = \frac{y+\frac{1}{3}}{\left(\frac{1}{3}\right)} = \frac{z-\frac{1}{3}}{\left(\frac{1}{6}\right)}$$

The direction ratios of this line are

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{6} \text{ i.e. } 3, 2, 1$$

Let \bar{b} be the vector parallel to this line.

$$\text{Then } \bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$$

The vector equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} is

$$\bar{r} = \bar{a} + \lambda\bar{b}, \text{ where } \lambda \text{ is a scalar}$$

\therefore the vector equation of the required line is

$$\bar{r} = (-\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k}).$$

The line passes through $(-1, -1, 2)$ and has direction ratios 3, 2, 1

\therefore the cartesian equations of the line are

$$\frac{x - (-1)}{3} = \frac{y - (-1)}{2} = \frac{z - 2}{1}$$

$$\text{i.e. } \frac{x + 1}{3} = \frac{y + 1}{2} = \frac{z - 2}{1}.$$

Miscellaneous Exercise 6 A | Q 19 | Page 208

Find the direction cosines of the lines

$$\bar{r} = \left(-2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda(2\hat{i} + 3\hat{j}).$$

Solution:

The line $\bar{r} = \left(-2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda(2\hat{i} + 3\hat{j})$ is parallel to $\bar{b} = 2\hat{i} + 3\hat{j}$.

\therefore direction ratios of the line are 2, 3, 0

∴ direction cosines of the line are

$$\frac{2}{\sqrt{2^2 + 3^2 + 0}}, \frac{3}{\sqrt{2^2 + 3^2 + 0}}, 0$$

$$\text{i.e. } \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0.$$

Miscellaneous Exercise 6 A | Q 20 | Page 208

Find the Cartesian equation of the line passing through the origin which is perpendicular to $x - 1 = y - 2 = z - 1$ and intersect the line

$$\frac{x - 1}{2} = \frac{y + 1}{3} = \frac{z - 1}{4}.$$

Solution: Let the required line have direction ratios a, b, c

Since the line passes through the origin, its cartesian equation are

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \quad \dots(1)$$

This line is perpendicular to the line

$x - 1 = y - 2 = z - 1$ whose direction ratios are $1, 1, 1$.

$$\therefore a + b + c = 0 \quad \dots(2)$$

The lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ intersect, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Applying this condition for the lines

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \text{ and } \frac{x - 1}{2} = \frac{y + 1}{3} = \frac{z - 1}{4} \text{ we get}$$

$$\begin{vmatrix} 1-0 & -1-0 & 1-0 \\ a & b & c \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\therefore 1(4b - 3c) + 1(4a - 2c) + 1(3a - 2b) = 0$$

$$\therefore 4b - 3c + 4a - 2c + 3a - 2b = 0$$

$$\therefore 7a + 2b - 5c = 0 \quad \dots(3)$$

From (2) and (3), we get

$$\frac{a}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 1 \\ -5 & 7 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 1 \\ 7 & 2 \end{vmatrix}}$$

$$\therefore \frac{a}{-7} = \frac{b}{12} = \frac{c}{-5}$$

\therefore the required line has direction ratios $-7, 12, -5$.

From (1), cartesian equation of required line are

$$\frac{x}{-7} = \frac{y}{12} = \frac{z}{-5}$$

$$\text{i.e. } \frac{x}{7} = \frac{y}{-12} = \frac{z}{5}.$$

Miscellaneous Exercise 6 A | Q 21 | Page 208

Find the vector equation of the line whose Cartesian equations are $y = 2$ and $4x - 3z + 5 = 0$.

Solution: $4x - 3z + 5 = 0$ can be written as

$$4x = 3z - 5 = 3\left(z - \frac{5}{3}\right)$$

$$\therefore \frac{4x}{12} = \frac{3\left(z - \frac{5}{3}\right)}{12}$$

$$\therefore \frac{x}{3} = \frac{z - \frac{5}{3}}{4}$$

\therefore the cartesian equation of the line are

$$\frac{x}{3} = \frac{z - \frac{5}{3}}{4}, y = 2.$$

This line passes through the point $A\left(0, 2, \frac{5}{3}\right)$ whose position vector is $\bar{a} = 2\hat{j} + \frac{5}{3}\hat{k}$

Also the line has direction ratio 3, 0, 4.

If \bar{b} is a vector parallel to the line, then $\bar{b} = 3\hat{i} + 4\hat{k}$

The vector equation of the line passing through

$A(\bar{a})$ and parallel to \bar{b} is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.

\therefore the vector equation of the required line is

$$\bar{r} = \left(2\hat{j} + \frac{5}{3}\hat{k}\right) + \lambda(3\hat{i} + 4\hat{k}).$$

Miscellaneous Exercise 6 A | Q 22 | Page 209

Find the coordinates of points on the line

$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{2}$ which are at the distance 3 unit from the base point $A(1, 2, 3)$.

Solution:

The cartesian equations of the line are

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{2} = \lambda \quad \dots(\text{Say})$$

The coordinates of any point on this line are given by

$$x = \lambda + 1, y = -2\lambda + 2, z = 2\lambda + 3$$

$$\text{Let } M(\lambda + 1, -2\lambda + 2, 2\lambda + 3) \quad \dots(1)$$

be the point on the line whose distance from A(1, 2, 3) is 3 units.

$$\therefore \sqrt{(\lambda + 1 - 1)^2 + (-2\lambda + 2 - 2)^2 + (2\lambda + 3 - 3)^2}$$

$$\therefore \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 3$$

$$\therefore \sqrt{9\lambda^2} = 3$$

$$\therefore 9\lambda^2 = 9$$

$$\therefore \lambda = \pm 1$$

$$\text{When } \lambda = 1, M = (1 + 1, -2 + 2, 2 + 3) \quad \dots[\text{By (1)}]$$

$$\text{i.e. } M = (2, 0, 5)$$

$$\text{When } \lambda = -1, M = (1 - 1, 2 + 2, -2 + 3) \quad \dots[\text{By (1)}]$$

$$\text{i.e. } M = (0, 4, 1)$$

Hence, the coordinates of the required points are (2, 0, 5) and (0, 4, 1).

MISCELLANEOUS EXERCISE 6 B [PAGES 223 - 225]

Miscellaneous Exercise 6 B | Q 1 | Page 223

Choose correct alternatives :

If the line $\frac{x}{3} = \frac{y}{4} = z$ is perpendicular to the line $\frac{x-1}{k} = \frac{y+2}{3} = \frac{z-3}{k-1}$, then the value of k is

1. 11/4
2. -11/4
3. 11/2
4. 4/11

Solution: - 11/4

Miscellaneous Exercise 6 B | Q 2 | Page 223

Choose correct alternatives :

The vector equation of line $2x - 1 = 3y + 2 = z - 2$ is

Options

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + 2\hat{k} \right) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$\bar{r} = \hat{i} - \hat{j} + (2\hat{i} + \hat{j} + \hat{k})$$

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \hat{j} \right) + \lambda(\hat{i} - 2\hat{j} + 6\hat{k})$$

$$\bar{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} - 2\hat{j} + 6\hat{k})$$

Solution:

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + 2\hat{k} \right) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$$

Miscellaneous Exercise 6 B | Q 3 | Page 223

Choose correct alternatives :

The direction ratios of the line which is perpendicular to the two lines

$$\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1} \text{ and } \frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2} \text{ are}$$

1. 4, 5, 7
2. 4, -5, 7
3. 4, -5, -7
4. -4, 5, 8

Solution: 4, 5, 7

Miscellaneous Exercise 6 B | Q 4 | Page 223

Choose correct alternatives :

The length of the perpendicular from (1, 6, 3) to the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

1. 3
2. $\sqrt{11}$
3. $\sqrt{13}$
4. 5

Solution: $\sqrt{13}$

Miscellaneous Exercise 6 B | Q 5 | Page 224

Choose correct alternatives :

The shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}) \text{ is}$$

1. $1/\sqrt{3}$
2. $1/\sqrt{2}$
3. $3/\sqrt{2}$
4. $\sqrt{3}/2$

Solution: $3/\sqrt{2}$

Miscellaneous Exercise 6 B | Q 6 | Page 224

Choose correct alternatives :

The lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k} \text{ and } \frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1} \text{ are}$$

coplanar if

1. $k = 1$ or -1
2. $k = 0$ or -3
3. $k = \pm 3$
4. $k = 0$ or -1



Solution: $k = 0$ or -3

Miscellaneous Exercise 6 B | Q 7 | Page 224

Choose correct alternatives :

The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{6}$ are

1. perpendicular
2. intersecting
3. skew
4. coincident

Solution: intersecting

Miscellaneous Exercise 6 B | Q 8 | Page 224

Choose correct alternatives :

Equation of X-axis is

1. $x = y = z$
2. $y = z$
3. $y = 0, z = 0$
4. $x = 0, y = 0$

Solution: $y = 0, z = 0$

Miscellaneous Exercise 6 B | Q 9 | Page 224

Choose correct alternatives :

The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

1. 45°
2. 30°
3. 0°
4. 90°

Solution: 90°

Miscellaneous Exercise 6 B | Q 10 | Page 224

Choose correct alternatives :

The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are

1. 2, 1, 6
2. 2, 1, -6
3. 2, -1, 6
4. -2, 1, 6

Solution: 2, 1, -6

Miscellaneous Exercise 6 B | Q 11 | Page 224

Choose correct alternatives :

The perpendicular distance of the plane $2x + 3y - z = k$ from the origin is $\sqrt{14}$ units, the value of k is

1. 14
2. 196
3. $2\sqrt{14}$
4. $\sqrt{14}/2$

Solution: 14

Miscellaneous Exercise 6 B | Q 12 | Page 224

Choose correct alternatives :

The angle between the planes

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) + 4 = 0 \text{ and } \vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) + 7 = 0 \text{ is}$$

Options

$$\frac{\pi}{2}$$

$$\frac{\pi}{3}$$

$$\cos^{-1}\left(\frac{3}{4}\right)$$

$$\cos^{-1}\left(\frac{9}{14}\right)$$

Solution:

$$\cos^{-1}\left(\frac{9}{14}\right)$$

Miscellaneous Exercise 6 B | Q 13 | Page 224

Choose correct alternatives :

If the planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} - \hat{j} + \mu\hat{k}) = 5$ are parallel, then the values of λ and μ are respectively

Options

$$\frac{1}{2}, -2$$

$$-\frac{1}{2}, 2$$

$$-\frac{1}{2}, -2$$

$$\frac{1}{2}, 2$$

Solution:

$$\frac{1}{2}, 2$$

Miscellaneous Exercise 6 B | Q 14 | Page 225

Choose correct alternatives :

The equation of the plane passing through (2, -1, 3) and making equal intercepts on the coordinate axes is

1. $x + y + z = 1$

2. $x + y + z = 2$

3. $x + y + z = 3$

4. $x + y + z = 4$



Solution: $x + y + z = 4$

Miscellaneous Exercise 6 B | Q 15 | Page 225

Choose correct alternatives :

Measure of angle between the plane $5x - 2y + 3z - 7 = 0$ and $15x - 6y + 9z + 5 = 0$ is

1. 0°
2. 30°
3. 45°
4. 90°

Solution: 0°

Miscellaneous Exercise 6 B | Q 16 | Page 225

Choose correct alternatives :

The direction cosines of the normal to the plane $2x - y + 2z = 3$ are

Options

$$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$$

$$\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$$

$$\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$$

$$\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Solution:

$$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$$

Miscellaneous Exercise 6 B | Q 17 | Page 225

Choose correct alternatives :

The equation of the plane passing through the points (1, -1, 1), (3, 2, 4) and parallel to Y-axis is :

1. $3x + 2z - 1 = 0$
2. $3x - 2z = 1$
3. $3x + 2z + 1 = 0$
4. $3x + 2z = 2$

Solution: $3x - 2z = 1$

Miscellaneous Exercise 6 B | Q 18 | Page 225

Choose correct alternatives :

The equation of the plane in which the line

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} \text{ lie, is}$$

1. $17x - 47y - 24z + 172 = 0$
2. $17x + 47y - 24z + 172 = 0$
3. $17x + 47y + 24z + 172 = 0$
4. $17x - 47y + 24z + 172 = 0$

Solution: $17x - 47y - 24z + 172 = 0$

Miscellaneous Exercise 6 B | Q 19 | Page 225

If the line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$ lies in the plane $3x - 14y + 6z + 49 = 0$, then the value of m is

1. 5
2. 3
3. 2
4. -5

Solution: 5

Miscellaneous Exercise 6 B | Q 20 | Page 225

Choose correct alternatives :

The foot of perpendicular drawn from the point (0,0,0) to the plane is (4, -2, -5) then the equation of the plane is

1. $4x + y + 5z = 14$
2. $4x - 2y - 5z = 45$
3. $x - 2y - 5z = 10$
4. $4x + y + 6z = 11$

Solution: $4x - 2y - 5z = 45$.

MISCELLANEOUS EXERCISE 6 B [PAGES 225 - 226]

Miscellaneous Exercise 6 B | Q 1 | Page 225

Solve the following :

Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector

$$2\hat{i} + \hat{j} + 2\hat{k}.$$

Solution:

If \hat{n} is a unit vector along the normal and p is the length of the perpendicular from origin to the plane, then the vector equation of the plane $\vec{r} \cdot \hat{n} = p$

Here, $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $p = 5$

$$\begin{aligned} \therefore |\vec{n}| &= \sqrt{2^2 + 1^2 + (2)^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{1}{3} (2\hat{i} + \hat{j} + 2\hat{k})$$

\therefore the vector equation of the required plane is

$$\vec{r} \cdot \left[\frac{1}{3} (2\hat{i} + \hat{j} + 2\hat{k}) \right] = 5$$

$$\text{i.e. } \vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 15.$$

Miscellaneous Exercise 6 B | Q 2 | Page 225

Solve the following :

Find the perpendicular distance of the origin from the plane $6x + 2y + 3z - 7 = 0$

Solution:

The distance of the point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is

$$d = 0 \text{ is } \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

\therefore the distance of the point $(1, 1, -1)$ from the plane $6x + 2y + 3z - 7 = 0$ is

$$\left| \frac{6(1) + 2(1) - 3(-1) + 7}{\sqrt{3^2 + 2^2 + (-1)^2}} \right|$$

$$= \left| \frac{6 + 2 + 3 + 7}{\sqrt{9 + 4 + 1}} \right|$$

$$= \frac{18}{\sqrt{14}}$$

$$= \frac{18}{\sqrt{14}}$$

$$= 1 \text{ units.}$$

Miscellaneous Exercise 6 B | Q 3 | Page 225

Solve the following :

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 3y + 6z = 49$.

Solution: The equation of the plane is $2x + 3y + 6z = 49$.

Dividing each term by

$$\sqrt{2^2 + 3^2 + (-6)^2}$$

$$= \sqrt{49}$$

$$= 7,$$

we get

$$\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = \frac{49}{7} = 7$$

This is the normal form of the equation of plane.

∴ the direction cosines of the perpendicular drawn from the origin to the plane are

$$l = \frac{2}{7}, m = \frac{3}{7}, n = \frac{6}{7}$$

and length of perpendicular from origin to the plane is $p = 7$.

∴ the coordinates of the foot of the perpendicular from the origin to the plane are

$$(lp, mp, np) \text{ i.e. } (2, 3, 6)$$

Miscellaneous Exercise 6 B | Q 4 | Page 225

Solve the following :

Reduce the equation $\vec{r} \cdot (6\hat{i} + 8\hat{j} + 24\hat{k}) = 13$ normal form and

hence find

- (i) the length of the perpendicular from the origin to the plane.
- (ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\vec{r} \cdot \hat{n} = p$ where \hat{n} is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.



Given plane is $\vec{r} \cdot (6\hat{i} + 8\hat{j} + 24\hat{k}) = 13$... (1)

$\vec{n} = 6\hat{i} + 8\hat{j} + 24\hat{k}$ is normal to the plane

$$\therefore |\vec{n}| = \sqrt{6^2 + 8^2 + 24^2} = \sqrt{76} = 13$$

Dividing both sides of (1) by 13, get

$$\vec{r} \cdot \left(\frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} \right) = \frac{76}{13}$$

$$\text{i.e. } \vec{r} \cdot \left(\frac{3}{13}\hat{i} + \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k} \right) = \frac{1}{2}$$

This is the normal form of the equation of plane.

Comparing with $\vec{r} \cdot \hat{n} = p$,

(i) the length of the perpendicular from the origin to plane is $\frac{1}{2}$.

(ii) direction cosines of the normal are $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$.

Miscellaneous Exercise 6 B | Q 5 | Page 226

Solve the following :

Find the vector equation of the plane passing through the points A(1, 92, 1), B(2, 91, 93) and C(0, 1, 5).

Solution: The vector equation of the plane passing through three non-collinear points

$$A(\vec{a}), B(\vec{b}) \text{ and } C(\vec{c}) \text{ is } \vec{r} \cdot (\overline{AB} \times \overline{AC}) = \vec{a} \cdot (\overline{AB} \times \overline{AC})$$

...(1)

$$\text{Here, } \vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}, \vec{c} = \hat{j} + 5\hat{k}$$

$$\therefore \overline{AB} = \bar{b} - \bar{a} = (2\hat{i} - \hat{j} - 3\hat{k}) - (\hat{i} - 2\hat{j} + \hat{k})$$

$$= \hat{i} + \hat{j} - 4\hat{k}$$

$$\overline{AC} = \bar{c} - \bar{a} = (\hat{j} + 5\hat{k}) - (\hat{i} - 2\hat{j} + \hat{k})$$

$$= \hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -4 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= (4 + 12)\hat{i} - (4 - 4)\hat{j} + (3 + 1)\hat{k}$$

$$= 16\hat{i} + 4\hat{k}$$

$$\text{Now, } \bar{a} \cdot (\overline{AB} \times \overline{AC}) = (\hat{i} - 2\hat{j} + \hat{k}) \cdot (16\hat{i} + 4\hat{k})$$

$$= (1)(16) + (-2)(0) + (1)(4) = 20$$

\therefore from (1), the vector equation of the required plane is

$$\bar{r} \cdot (16\hat{i} + 4\hat{k}) = 20.$$

Miscellaneous Exercise 6 B | Q 6 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(1, -2, 3) and direction ratios of whose normal are 0, 2, 0.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction ratios of whose normal are a, b, c , is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

\therefore the cartesian equation of the required plane is

$$0(x + 1) + 2(y + 2) + 5(z - 3) = 0$$

i.e. $0 + 2y - 4 + 10z - 15 = 0$

i.e. $y + 2 = 0$.

Miscellaneous Exercise 6 B | Q 7 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(7, 8, 6) and parallel to the plane $\vec{r} \cdot (6\hat{i} + 8\hat{j} + 7\hat{k}) = 0$.

Solution: The cartesian equation of the plane

$$\vec{r} \cdot (6\hat{i} + 8\hat{j} + 7\hat{k}) = 0 \text{ is } 6x + 8y + 7z = 0$$

The required plane is parallel to it

\therefore its cartesian equation is

$$6x + 8y + 7z = p \quad \dots(1)$$

A(7, 8, 6) lies on it and hence satisfies its equation

$$\therefore (6)(7) + (8)(8) + (7)(6) = p$$

$$\text{i.e., } p = 42 + 64 + 42 = 148.$$

\therefore from (1), the cartesian equation of the required plane is $6x + 8y + 7z = 148$.

Miscellaneous Exercise 6 B | Q 8 | Page 226

Solve the following :

The foot of the perpendicular drawn from the origin to a plane is M(1, 2, 0). Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through A(\vec{a}) and perpendicular to \vec{n} is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

M(1, 2, 0) is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.

If \vec{m} is the position vector of M, then $\vec{m} = \hat{i}$.



Normal to the plane is

$$\bar{n} = \overline{OM} = \hat{i}$$

$$\bar{m} \cdot \bar{n} = \hat{i} \cdot \hat{i} = 5$$

\therefore the vector equation of the required plane is

$$\bar{r} \cdot (\hat{i} + 2\hat{j}) = 5.$$

Miscellaneous Exercise 6 B | Q 9 | Page 226

Solve the following :

A plane makes non zero intercepts a, b, c on the coordinate axes.

Show that the vector equation of the plane is

$$\bar{r} \cdot (bc\hat{i} + ca\hat{j} + ab\hat{k}) = abc.$$

Solution:

The vector equation of the plane passing through

$A(\bar{a}), B(\bar{b}), C(\bar{c})$, where A, B, C are non-collinear is

$$\bar{r} \cdot (\overline{AB} \times \overline{AC}) = \bar{a} \cdot (\overline{AB} \times \overline{AC}) \quad \dots(1)$$

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

\therefore it passes through the three non-collinear points $A = (1, 0, 0)$, $B = (0, 1, 0)$, $C = (0, 0, 1)$

$$\therefore \bar{a} = \hat{i}, \bar{b} = \hat{j}, \bar{c} = \hat{k}$$

$$\overline{AB} = \bar{b} - \bar{a} = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$$

$$\therefore \overline{AC} = \bar{c} - \bar{a} = \hat{k} - \hat{i} = -\hat{i} + \hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= (1 - 0)\hat{i} - (-1 - 0)\hat{j} + (0 + 1)\hat{k}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

Also,

$$\begin{aligned} \bar{a} \cdot (\overline{AB} \times \overline{AC}) \\ = \hat{i} \cdot (\hat{i} + \hat{j} + \hat{k}) \end{aligned}$$

$$= 1 \times 1 + 0 \times 1 + 0 \times 1$$

$$= 1$$

\therefore from (1) the vector equation of the required plane is

$$\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1.$$

Miscellaneous Exercise 6 B | Q 10 | Page 226

Solve the following :

Find the vector equation of the plane passing through the point $A(-2, 3, 5)$ and parallel to the vectors $4\hat{i} + 3\hat{k}$ and $\hat{i} + \hat{j}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors \bar{b} and \bar{c} is

$$\bar{r} \cdot (\bar{b} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c}) \quad \dots(1)$$

Here, $\bar{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$

$\bar{b} = 4\hat{i} + 3\hat{k}$,

$\bar{c} = \hat{i} + \hat{j}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= (-1 - 3)\hat{i} - (4 - 3)\hat{j} + (3 + 1)\hat{k}$$

$$= -4\hat{i} - \hat{j} + 4\hat{k}$$

$$\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (-2\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (-4\hat{i} - \hat{j} + 4\hat{k})$$

$$= (-2)(-4) + (3)(-1) + (5)(4)$$

$$= 8 - 3 + 20$$

$$= 25$$

\therefore From (1), the vector equation of the required plane is

$$\bar{r} \cdot (-3\hat{i} - \hat{j} + 4\hat{k}) = 25.$$

Miscellaneous Exercise 6 B | Q 11 | Page 226

Solve the following :

Find the cartesian equation of the plane

$$\bar{r} = \lambda(\hat{i} + \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k}).$$

Solution:

The equation $\bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$ represents a plane passing through a point having position vector \bar{a} and parallel to vectors \bar{b} and \bar{c} .

Here,

$$\bar{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}},$$

$$\bar{\mathbf{c}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$$\therefore \bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= (3 + 2)\hat{\mathbf{i}} - (3 - 1)\hat{\mathbf{j}} + (-2 - 1)\hat{\mathbf{k}}$$

$$= 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$= \bar{\mathbf{a}}$$

Also,

$$\bar{\mathbf{a}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$$

$$= \bar{\mathbf{a}} \cdot \bar{\mathbf{a}} = |\bar{\mathbf{a}}|^2$$

$$= (5)^2 + (4)^2 + (0)^2$$

$$= 0$$

The vector equation of the plane passing through A($\bar{\mathbf{a}}$) and parallel to $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ is

$$\bar{\mathbf{r}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}}) = \bar{\mathbf{a}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$$

\therefore the vector equation of the given plane is

$$\bar{\mathbf{r}} \cdot (5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$$

If $\bar{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then this equation becomes

$$(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$$

$$\therefore 5x - 4y + z = 0.$$

This is the cartesian equation of the required plane.

Miscellaneous Exercise 6 B | Q 12 | Page 226

Solve the following :

Find the cartesian equations of the planes which pass through A(1, 2, 3), B(3, 2, 1) and make equal intercepts on the coordinate axes.

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the cartesian equation of the plane is $ax + by + cz = 0$ (1)

(1, 2, 3) and (3, 2, 1) lie on the plane.

$$\therefore a + 2b + 3c = 0 \text{ and } 3a + 2b + c = 0$$

$$\therefore \frac{a}{\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{a}{-4} = \frac{b}{8} = \frac{c}{-4}$$

$$\text{i.e. } \frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$$

$\therefore a, b, c$ are proportional to 1, -2, 1

\therefore from (1), the required cartesian equation is $x - 2y + z = 0$

Case 2 : Let the plane make non zero intercept p on each axis.

$$\text{then its equation is } \frac{x}{p} + \frac{y}{p} + \frac{z}{p} = 1$$

$$\text{i.e. } x + y + z = p \quad \dots (2)$$

Since this plane passes through (1, 2, 3) and (3, 2, 1)

$$\therefore 1 + 2 + 3 = p \text{ and } 3 + 2 + 1 = p$$

$$\therefore p = 6$$

\therefore from (2), the required cartesian equation is

$$x + y + z = 6$$

Hence, the cartesian equations of required planes are

$$x + y + z = 6 \text{ and } x - 2y + z = 0.$$

Miscellaneous Exercise 6 B | Q 13 | Page 226



Solve the following :

Find the vector equation of the plane which makes equal non zero intercepts on the coordinate axes and passes through (1, 1, 1).

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the vector equation of the plane is $ax + by + cz = 0$ (1)

(1, 1, 1) lie on the plane.

$$\therefore 1a + 1b + 1c = 0$$

$$\therefore \frac{\hat{i}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{j}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{k}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\therefore \frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$$

$$\text{i.e. } \frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$$

$\therefore \hat{i}, \hat{j}, \hat{k}$ are proportional to 1, 1, 1

\therefore from (1), the required cartesian equation is $x - y + z = 0$

Case 2 : Let the plane make non zero intercept p on each axis.

$$\text{then its equation is } \frac{\hat{i}}{p} + \frac{\hat{j}}{p} + \frac{\hat{k}}{p} = 1$$

$$\text{i.e. } \hat{i} + \hat{j} + \hat{k} = p \quad \dots (2)$$

Since this plane passes through (1, 1, 1)

$$\therefore 1 + 1 + 1 = p$$

$$\therefore p = 3$$

\therefore from (2), the required cartesian equation is $\hat{i} + \hat{j} + \hat{k} = 3$

Hence, the cartesian equations of required planes are

$$\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$$

Miscellaneous Exercise 6 B | Q 14 | Page 226

Solve the following :

Find the angle between the planes $\bar{r} \cdot (-2\hat{i} + \hat{j} + 2\hat{k}) = 17$ and $\bar{r} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 71$.

Solution:

The acute angle between the planes

$\bar{r} \cdot \bar{n}_1 = d_1$ and $\bar{r} \cdot \bar{n}_2 = d_2$ is given by

$$\cos \theta = \left| \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1| |\bar{n}_2|} \right| \quad \dots(1)$$

Here,

$$\bar{n}_1 = -2\hat{i} + \hat{j} + 2\hat{k},$$

$$\bar{n}_2 = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore \bar{n}_1 \cdot \bar{n}_2$$

$$= (2\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k})$$

$$= (1)(2) + (1)(1) + (2)(1)$$

$$= 2 + 1 + 2$$

$$= 5$$

Also,

$$|\bar{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$|\bar{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

∴ from (1), we have

$$\cos \theta = \left| \frac{3}{\sqrt{6}\sqrt{6}} \right|$$

$$= \frac{3}{6}$$

$$= \frac{1}{2} \cos 90^\circ$$

$$\therefore \theta = 90^\circ.$$

Miscellaneous Exercise 6 B | Q 15 | Page 226

Solve the following :

Find the acute angle between the line $\bar{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane $\bar{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 23$.

Solution:

The acute angle θ between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r} \cdot \bar{n} = d$ is given by

$$\sin \theta = \left| \frac{\bar{b} \cdot \bar{n}}{|\bar{b}| |\bar{n}|} \right| \quad \dots(1)$$

$$\text{Here, } \bar{b} = \hat{i} - \hat{j} + \hat{k}, \bar{n} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore \bar{\mathbf{b}} \cdot \bar{\mathbf{n}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (2)(2) + (3)(-1) + (-6)(1)$$

$$= 4 - 3 - 6$$

$$= -5$$

$$\text{Also, } |\bar{\mathbf{b}}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{2} = 1$$

$$|\bar{\mathbf{n}}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4}$$

\therefore from (1), we have

$$\sin \theta = \left| \frac{2\sqrt{2}}{-3} \right| = \frac{2\sqrt{2}}{3}$$

$$\therefore \theta = \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right).$$

Miscellaneous Exercise 6 B | Q 16 | Page 226

Show that the line $\bar{\mathbf{r}} = (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ and $\bar{\mathbf{r}} = (2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ are coplanar. Find the equation of the plane determined by them.

Solution:

The lines $\bar{\mathbf{r}} = \bar{\mathbf{a}}_1 + \lambda_1 \bar{\mathbf{b}}_1$ and $\bar{\mathbf{r}} = \bar{\mathbf{a}}_2 + \lambda_2 \bar{\mathbf{b}}_2$ are coplanar if

$$\bar{\mathbf{a}}_1 \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2) = \bar{\mathbf{a}}_2 \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2)$$

$$\text{Here } \bar{\mathbf{a}}_1 = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \bar{\mathbf{a}}_2 = 2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}},$$

$$\bar{\mathbf{b}}_1 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \bar{\mathbf{b}}_2 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$$\therefore \bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1 = (2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

$$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= (8 - 9)\hat{i} - (4 - 6)\hat{j} + (3 - 4)\hat{k}$$

$$= -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})$$

$$= 0(-1) + 2(2) + (-3)(-1)$$

$$= 0 + 4 + 3$$

$$= 7$$

$$\text{and } \bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2) = (2\hat{j} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})$$

$$= 2(-1) + 6(2) + 3(-1)$$

$$= -2 + 12 - 3$$

$$= 7$$

$$\therefore \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_2 \cdot (\bar{b}_1 \times \bar{b}_2)$$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$$\therefore \bar{r} \cdot (\bar{b}_1 \times \bar{b}_2) = \bar{a}_1 \cdot (\bar{b}_1 \times \bar{b}_2)$$

$$\text{i.e. } \bar{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$$

Hence, the given lines are coplanar and the equation of the plane determined by these lines is

$$\bar{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7.$$

Miscellaneous Exercise 6 B | Q 17 | Page 226

Solve the following :

Find the distance of the point $3\hat{i} + 3\hat{j} + \hat{k}$ from the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = 21$.

Solution:

The distance of the point $A(\vec{a})$ from the plane $\vec{r} \cdot \vec{n} = p$ is given by $d = \frac{|\vec{a} \cdot \vec{n} - p|}{|\vec{n}|}$... (1)

Here, $\vec{a} = 3\hat{i} + 3\hat{j} + \hat{k}$, $\vec{n} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $p = 21$

$$\therefore \vec{a} \cdot \vec{n} = (3\hat{i} + 3\hat{j} + \hat{k}) \cdot (2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$= (3)(2) + (3)(3) + (1)(-6)$$

$$= 6 + 6 - 6$$

$$= 6$$

$$\text{Also, } |\vec{n}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{-12} = 0$$

\therefore from (1), the required distance

$$= \frac{|-12 - 21|}{12}$$

$$= 0 \text{ units.}$$

Miscellaneous Exercise 6 B | Q 18 | Page 226

Solve the following :

Find the distance of the point $(13, 13, -13)$ from the plane $3x + 4y - 12z = 0$.



Solution:

The distance of the point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is

$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

\therefore the distance of the point $(1, 1, -1)$ from the plane $3x + 4y - 12z$

$$= 0 \text{ is } \left| \frac{3(1) + 4(1 - 12(-1))}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$$

$$= \left| \frac{3 + 4 + 12}{\sqrt{9 + 16 + 144}} \right|$$

$$= \frac{19}{\sqrt{169}}$$

$$= \frac{19}{13}$$

= 19units.

Miscellaneous Exercise 6 B | Q 19 | Page 226

Solve the following :

Find the vector equation of the plane passing through the origin and containing the line $\vec{r} = (\hat{i} + 4\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$.

Solution:

The vector equation of the plane passing through $A(\vec{a})$ and perpendicular to the vector \vec{n} is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \quad \dots(1)$

We can take $\vec{a} = \vec{0}$ since the plane passes through the origin.

The point M with position vector $\vec{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the line and hence it lies on the plane.

$\therefore \vec{OM} = \vec{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the plane.

The plane contains the given line which is parallel to $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Let \vec{n} be normal to the plane. Then \vec{n} is perpendicular to \vec{OM} as well as \vec{b}

$$\begin{aligned}\therefore \vec{n} &= \vec{OM} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 1 \\ 1 & 2 & 1 \end{vmatrix} \\ &= (4 - 2)\hat{i} - (1 - 1)\hat{j} + (2 - 4)\hat{k} \\ &= 2\hat{i} - 2\hat{k}\end{aligned}$$

\therefore from (1), the vector equation of the required plane is

$$\vec{r} \cdot (2\hat{i} - 2\hat{k}) = \vec{0} \cdot \vec{n} = 0$$

$$\text{i.e. } \vec{r} \cdot (\hat{i} - \hat{k}) = 0.$$

Miscellaneous Exercise 6 B | Q 20 | Page 226

Solve the following :

Find the vector equation of the plane which bisects the segment joining A(2, 3, 6) and B(4, 3, -2) at right angle.

Solution:

The vector equation of the plane passing through A(\vec{a}) and perpendicular to the vector \vec{n} is $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \quad \dots(1)$

The position vectors \bar{a} and \bar{b} of the given points A and B are

$$\bar{a} = 2\hat{i} + 3\hat{j} + 6\hat{k} \text{ and } \bar{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$$

If M is the midpoint of segment AB, the position vector \bar{m} of M is given by

$$\begin{aligned}\bar{m} &= \frac{\bar{a} + \bar{b}}{2} \\&= \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) + (4\hat{i} + 3\hat{j} - 2\hat{k})}{2} \\&= \frac{6\hat{i} + 6\hat{j} + 4\hat{k}}{2} \\&= 3\hat{i} + 3\hat{j} + 2\hat{k}\end{aligned}$$

The plane passes through M(\bar{m}).

AB is perpendicular to the plane

If \bar{n} is normal to the plane, then $\bar{n} = \overline{AB}$

$$\therefore \bar{n} = \bar{b} - \bar{a} = (4\hat{i} + 3\hat{j} - 2\hat{k}) - (2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$= 2\hat{i} - 8\hat{k}$$

$$\therefore \bar{m} \cdot \bar{n} = (3\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 8\hat{k})$$

$$= (3)(2) + (3)(0) + (2)(-8)$$

$$= 6 + 0 - 16$$

$$= -10$$

\therefore from (1), the vector equation of the required plane is

$$\bar{r} \cdot \bar{n} = \bar{m} \cdot \bar{n}$$

$$\text{i.e. } \vec{r} \cdot (2\hat{i} - 8\hat{k}) = -10$$

$$\text{i.e. } \vec{r} \cdot (\hat{i} - 4\hat{k}) = -5.$$

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Solve the following :

Show that the lines $x = y, z = 0$ and $x + y = 0, z = 0$ intersect each other. Find the vector equation of the plane determined by them.

Solution:

Given lines are $x = y, z = 0$ and $x + y = 0, z = 0$.

It is clear that $(0, 0, 0)$ satisfies both the equations.

\therefore the lines intersect at O whose position vector is $\vec{0}$

Since $z = 0$ for both the lines, both the lines lie in XY-plane.

Hence, we have to find equation of XY-plane.

Z-axis is perpendicular to XY-plane.

\therefore normal to XY plane is \hat{k} .

$O(\vec{0})$ lies on the plane.

By using $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$, vector equation of the required plane is

$$\vec{r} \cdot \hat{k} = \vec{0} \cdot \hat{k}$$

$$\text{i.e. } \vec{r} \cdot \hat{k} = 0.$$

Hence, the given lines intersect each other and the vector equation of the plane determined by them is $\vec{r} \cdot \hat{k} = 0$.